

S21CAF – NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S21CAF evaluates the Jacobian elliptic functions sn, cn and dn.

2 Specification

```
SUBROUTINE S21CAF(U, M, SN, CN, DN, IFAIL)
  INTEGER          IFAIL
  real            U, M, SN, CN, DN
```

3 Description

This routine evaluates the Jacobian elliptic functions of argument u and parameter m ,

$$\begin{aligned} \operatorname{sn}(u|m) &= \sin \phi, \\ \operatorname{cn}(u|m) &= \cos \phi, \\ \operatorname{dn}(u|m) &= \sqrt{1 - m \sin^2 \phi}, \end{aligned}$$

where ϕ , called the *amplitude* of u , is defined by the integral

$$u = \int_0^\phi \frac{d\theta}{\sqrt{1 - m \sin^2 \theta}}.$$

The elliptic functions are sometimes written simply as snu , cnu and dnu , avoiding explicit reference to the parameter m .

Another nine elliptic functions may be computed via the formulae

$$\begin{aligned} \operatorname{cdu} &= \operatorname{cnu}/\operatorname{dnu} \\ \operatorname{sdu} &= \operatorname{snu}/\operatorname{dnu} \\ \operatorname{ndu} &= 1/\operatorname{dnu} \\ \operatorname{dcu} &= \operatorname{dnu}/\operatorname{cnu} \\ \operatorname{ncu} &= 1/\operatorname{cnu} \\ \operatorname{scu} &= \operatorname{snu}/\operatorname{cnu} \\ \operatorname{nsu} &= 1/\operatorname{snu} \\ \operatorname{dsu} &= \operatorname{dnu}/\operatorname{snu} \\ \operatorname{csu} &= \operatorname{cnu}/\operatorname{snu} \end{aligned}$$

(see Abramowitz and Stegun [1]).

S21CAF is based on a procedure given by Bulirsch [2], and uses the process of the arithmetic-geometric mean ([1], 16.9). Constraints are placed on the values of u and m in order to avoid the possibility of machine overflow.

4 References

- [1] Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)
- [2] Bulirsch R (1965) Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Parameters

- 1: U — *real* *Input*
 2: M — *real* *Input*

On entry: the argument u and the parameter m of the functions, respectively.

Constraints:

$$\begin{aligned} \text{ABS}(U) &\leq \sqrt{\lambda}, \text{ where } \lambda = 1/\text{X02AMF}, \\ \text{ABS}(M) &\leq \sqrt{\lambda} \text{ if } \text{ABS}(U) < 1/\sqrt{\lambda}. \end{aligned}$$

- 3: SN — *real* *Output*
 4: CN — *real* *Output*
 5: DN — *real* *Output*

On exit: the values of the functions $\text{sn}u$, $\text{cn}u$ and $\text{dn}u$, respectively.

- 6: IFAIL — INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

If on exit IFAIL \neq 0, then S21CAF returns with the value 0.0 for sn, cn and dn.

Errors detected by the routine:

IFAIL = 1

On entry, $\text{ABS}(U) > \sqrt{\lambda}$, where $\lambda = 1/\text{X02AMF}$.

IFAIL = 2

On entry, $\text{ABS}(M) > \sqrt{\lambda}$ and $\text{ABS}(U) < 1/\sqrt{\lambda}$.

7 Accuracy

In principle the routine is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the Fortran intrinsic functions for elementary functions such as SIN and COS.

8 Further Comments

None.

9 Example

The following program reads values of the argument u and parameter m from a file, evaluates the function and prints the results.

9.1 Program Text

Note. The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      S21CAF Example Program Text
*      Mark 15 Release. MAG Copyright 1991
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real             CN, DN, M, SN, U
      INTEGER          IFAIL
*      .. External Subroutines ..
      EXTERNAL         S21CAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S21CAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*)
+ '          U          M          SN          CN          DN'
20 READ (NIN,*,END=40) U, M
*
      IFAIL = 0
      CALL S21CAF(U,M,SN,CN,DN,IFAIL)
*
      WRITE (NOUT,99999) U, M, SN, CN, DN
      GO TO 20
40 STOP
*
99999 FORMAT (3X,5e13.4)
      END

```

9.2 Program Data

```

S21CAF Example Program Data
 0.2  0.3
 5.0 -1.0
-0.5 -0.1
10.0 11.0

```

9.3 Program Results

S21CAF Example Program Results

U	M	SN	CN	DN
0.2000E+00	0.3000E+00	0.1983E+00	0.9801E+00	0.9941E+00
0.5000E+01	-0.1000E+01	-0.2440E+00	0.9698E+00	0.1029E+01
-0.5000E+00	-0.1000E+00	-0.4812E+00	0.8766E+00	0.1012E+01
0.1000E+02	0.1100E+02	0.2512E+00	0.9679E+00	0.5528E+00